International Journal of Theoretical Physics, Vol. 34, No. 9, 1995

Stark Shift Contribution to Field Statistics in a Generalized Jaynes–Cummings Model

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Received January 28, 1995

The nonlinear and multiphoton interaction between a single two-level atom and two modes of radiation is studied in a generalized Jaynes–Cummings model. An intensity-dependent level shift is considered. The time evolution operator is obtained. The detuning has a photon number dependence. Different statistical aspects pertaining to either the atom or the fields are calculated. The dipole moment, the dipole–dipole correlation function, as well as the transient spectrum are obtained.

1. INTRODUCTION

The interaction of a single two-level atom and the radiation field [Jaynes and Cummings (1963) model, JCM] has been studied extensively (Cummings, 1965; Eberly *et al.*, 1980; Knight and Radmore, 1982a,b; Narozhny *et al.*, 1981; Yoo and Eberly, 1985; Tavis and Cummings, 1969). Various models have been used to discuss many different phenomena (Allen and Eberly, 1975; Sargent *et al.*, 1974; Shumovsky *et al.*, 1985). A number of generalized JCM models have been investigated (Abdalla *et al.*, 1990, 1991; Abdel-Hafez *et al.*, 1987; Obada and Abdel-Hafez, 1986; Ackerhalt and Rzazeuski, 1975; Buck and Sukumar, 1981, 1984; Sukumar and Buck, 1984; Kosierowski, 1986; Kosierowski and Shumovsky, 1987; Kochetov, 1987; Singh, 1982).

In this paper, a generalized JCM is studied; the interaction is multiphoton and nonlinear. Also, the Stark shift is considered. The constants of motion are obtained. The time evolution operator is calculated and used to compute the density matrix. Consequently, different distributions have been assumed

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0020-7748/95/0900-1877$07.50/0

for the initial photon states. We note that the detuning parameter depends on the photon numbers. Finally, the transient spectrum is obtained.

2. THE TIME EVOLUTION OPERATOR

We consider the system of a single two-level atom interacting with two modes of radiation, where the interaction is multiphoton and nonlinear. This model differs from those considered before (Abdalla *et al.*, 1991) by the addition of the term $\Sigma_1^2 \alpha_j \hat{n}_j \hat{S}_{jj}$, which represents an intensity-dependent level shift (Stark shift).

The Hamiltonian that describes this model in the rotating wave approximation (RWA) is given by

$$\hat{H} = \sum_1^2 w_j \hat{n}_j + \sum_1^2 \Omega_j \hat{S}_{jj} + \sum_1^2 \alpha_j \hat{n}_j \hat{S}_{jj} + \lambda(\hat{S}_{12}(\hat{a}_2^+)^{K_2}\hat{a}_1^{k_1} + (\hat{a}_1^+)^{k_1}\hat{a}_2^{k_2}\hat{S}_{21}) \tag{2.1}$$

where w_j are the field frequencies and Ω_j the level energies, while $\hat{a}_j^+$ and $\hat{a}_j$ are the boson operators for the quantized field, which obey the commutation relation

$$[\hat{a}_i, \hat{a}_j^+] = \delta_{ij} \tag{2.2}$$

The $\hat{S}_{ij}$ operators are the generators of the group $U(2)$. They describe the atom and satisfy

$$[\hat{S}_{ij}, \hat{S}_{kl}] = \hat{S}_{il}\delta_{kj} - \hat{S}_{kj}\delta_{il} \tag{2.3}$$

we note that the original Jaynes–Cummings (1963) model is given if we put $\alpha_j = 0, k_2 = 0$, and $k_1 = 1$ and the generalized model obtained above (Abdalla *et al.*, 1991) has $\alpha_j = 0$, i.e., this model is a more general one. Here λ is the coupling constant.

By using the Heisenberg equation of motion for the operators $\hat{n}_j = a_j^+ a_j$ and $\hat{S}_{jj}$, we can deduce the following constants of motion:

$$\hat{N}_1 = \hat{n}_1 + \frac{1}{2} k_1(\hat{S}_{11} - \hat{S}_{22}) \tag{2.4a}$$

$$\hat{N}_2 = \hat{n}_2 - \frac{1}{2} k_2(\hat{S}_{11} - \hat{S}_{22}) \tag{2.4b}$$

Thus, the Hamiltonian (2.1) becomes

$$\hat{H} = \sum_{1}^{2}\left(w_j + \frac{1}{2}\alpha_j\right)\hat{N}_j + \frac{1}{2}\left(\Omega_1 + \Omega_2 - \frac{1}{2}\alpha_1 k_1 - \frac{1}{2}\alpha_2 k_2\right)\hat{I} + \hat{C} \tag{2.5}$$

where

$$\hat{C} = \hat{q} + \hat{D} \tag{2.6a}$$

$$\hat{D} = \lambda(\hat{S}_{12}(\hat{a}_2^+)^{K_2}\hat{a}_1^{k_1} + (\hat{a}_1^+)^{k_1}\hat{a}_2^{k_2}\hat{S}_{21}) \tag{2.6b}$$

$$2\hat{q} = \hat{E}_1\hat{S}_{11} - \hat{E}_2\hat{S}_{22} \tag{2.6c}$$

with the new detuning parameter $\hat{E}_j$ which depends on photon numbers as follows:

$$\hat{E}_1 = \Delta - \frac{1}{2}\alpha_1 k_1 + \frac{1}{2}\alpha_2 k_2 + \alpha_1\left(n_1 + \frac{1}{2}k_1\right) - \alpha_2\left(n_2 - \frac{1}{2}k_2\right) \tag{2.7a}$$

$$\hat{E}_2 = \Delta - \frac{1}{2}\alpha_1 k_1 + \frac{1}{2}\alpha_2 k_2 + \alpha_1\left(n_1 - \frac{1}{2}k_1\right) - \alpha_2\left(n_2 + \frac{1}{2}k_2\right) \tag{2.7b}$$

with

$$\Delta = \Omega_1 - \Omega_2 + k_2 w_2 - k_1 w_1 \tag{2.8}$$

If we put $\alpha_j = 0$, we have $\hat{E}_j = \Delta$, which is the usual detuning parameter.

We note that the $\hat{E}_j$ operators satisfy the following relation:

$$\hat{E}_1\hat{a}_1^{k_1}(\hat{a}_2^+)^{k_2} = \hat{a}_1^{k_1}(\hat{a}_2^+)^{k_2}\hat{E}_2 \tag{2.9}$$

We can show that

$$\hat{q}\hat{D} + \hat{D}\hat{q} = 0 \tag{2.10}$$

It is easy to show that $[\hat{C}, \hat{N}] = 0$ and hence each of them commutes with $\hat{H}$, i.e., $\hat{N}$ and $\hat{C}$ are constants of motion.

Now we consider the time evolution operator $\hat{U}(t)$, which is defined as follows:

$$\begin{aligned}
\hat{U}(t) &= \exp\{-i\hat{H}t\} \\
&= \exp\left\{-i\sum_{1}^{2}(w_j + \alpha_j)\hat{N}_j t\right\} \\
&\quad \times \exp\left\{-\frac{1}{2}i\left(\Omega_1 + \Omega_2 - \frac{1}{2}\alpha_1 k_1 - \frac{1}{2}\alpha_2 k_2\right)t\right\} \\
&\quad \times \exp\{i\hat{C}t\}
\end{aligned} \tag{2.11}$$

taking into account the fact that $[\hat{C}, \hat{N}] = 0$.

After some manipulations, we can show that the time evolution operator is given by

$$U(t) = \exp\left\{-\frac{1}{2}\left(\Omega_1 + \Omega_2 - \frac{1}{2}\alpha_1 k_1 - \frac{1}{2}\alpha_2 k_2\right)t\right\}$$
$$\times \begin{bmatrix} \exp(-iz_1 t) & 0 \\ 0 & \exp(-iz_2 t) \end{bmatrix} \exp(-i\hat{C}t) \tag{2.12}$$

where

$$Z_1 = \left(w_1 + \frac{1}{2}\alpha_1\right)\left(n_1 + \frac{1}{2}k_1\right) + \left(w_2 + \frac{1}{2}\alpha_2\right)\left(n_2 - \frac{1}{2}k_2\right) \tag{2.13a}$$

$$Z_2 = \left(w_1 + \frac{1}{2}\alpha_1\right)\left(n_1 - \frac{1}{2}k_1\right) + \left(w_2 + \frac{1}{2}\alpha_2\right)\left(n_2 + \frac{1}{2}k_2\right) \tag{2.13b}$$

and

$$\exp(-i\hat{C}t) = \begin{bmatrix} \cos\hat{\mu}_1 t - \frac{i\hat{E}_1}{2}\frac{\sin\hat{\mu}_1 t}{\hat{\mu}_1} & -i\lambda\frac{\sin\hat{\mu}_1 t}{\hat{\mu}_1}(\hat{a}_2^+)^{k_2}\hat{a}_1^{k_1} \\ -i\lambda\frac{\sin\hat{\mu}_2 t}{\hat{\mu}_2}(\hat{a}_1^+)^{k_1}\hat{a}_2^{k_2} & \cos\hat{\mu}_2 t + \frac{i\hat{E}_2}{2}\frac{\sin\hat{\mu}_2 t}{\hat{\mu}_2} \end{bmatrix} \tag{2.13c}$$

with

$$\hat{\mu}_j^2 = \frac{\hat{E}_j^2}{4} + \lambda^2\frac{(\hat{n}_j + k_j)!}{\hat{n}_j!}\frac{\hat{n}_i!}{(\hat{n}_i - k_i)!} = \frac{\hat{E}_j^2}{4} + \hat{\nu}_j \tag{2.14}$$

The μ_j are the generalized Rabi frequencies in this case and $\hat{\nu}_j$ are the Rabi frequencies for zero detuning.

It is easy to show that $U(t)U^+(t) = I$. Once the time evolution operator is known, the dynamical behavior of any operator can be determined through the relation

$$\hat{O}(t) = \hat{U}^+(t)O(0)U(t) \tag{2.15}$$

where $\hat{O}(0)$ is the initial value operator.

3. SOME STATISTICAL ASPECTS

For the statistical averages we have

$$\langle \hat{O}(t) \rangle = \mathrm{tr}[\hat{\rho}(t)\hat{O}(0)] \tag{3.1}$$

where the density matrix is

$$\hat{\rho}(t) = \hat{U}(t)\rho(0)\hat{U}^{+}(t) \tag{3.2}$$

and the probability distribution function is obtained from the expectation value of the field density matrix

$$\hat{\rho}_F(t) = \mathrm{Tr}_A \hat{\rho}(t) \tag{3.3}$$

between the photon number states.

Here we assume that at $t = 0$, the density matrix takes the following form:

$$\hat{\rho}(0) = \hat{\rho}_A(0) \otimes \hat{\rho}_F(0) \tag{3.4}$$

When the atom is prepared to be in its pure ground state, the initial value for the density matrix $\hat{\rho}(0)$ takes the form

$$\hat{\rho}^{\mathrm{gr}}(0) = \hat{\rho}_F \hat{S}_{11}(0) \tag{3.5a}$$

while the density matrix $\hat{\rho}^{\mathrm{ex}}(0)$, when the atom is prepared to be in its pure excited state, is given by

$$\hat{\rho}^{\mathrm{ex}}(0) = \hat{\rho}_F \hat{S}_{22}(0) \tag{3.5b}$$

From the density matrix for the field at any time $t > 0$ given by (3.3), the probability distribution function of finding n_i photons in the mode i at $t > 0$ as mentioned above is given by

$$P(n_1, n_2, t) = \langle n_1, n_2 | \hat{\rho}_F(t) | n_1, n_2 \rangle \tag{3.6}$$

By using equations (3.1)–(3.6) we can obtain some statistical quantities.

3.1. The Atom in Its Ground State

The field density matrix is given generally by

$$\hat{\rho}_F(0) = \sum_{\substack{m_1, m_2 \\ m_1', m_2'}} |m_1, m_2\rangle\langle m_1', m_2'| \rho^F_{m_1 m_2, m_1' m_2'} \tag{3.7}$$

Making use of the above field density matrix (3.7) in the expression for the density matrix with the ground state (3.5), we obtain the expression for the probability distribution function

$$P^{gr}(n_1 n_2, t) = \nu_1 \frac{\sin^2 \mu_1 t}{\mu_1^2} P(n_1 + k_1, n_2 - k_2) + \left(\cos^2 \mu_2 t + \frac{E_2^2}{4\mu_2^2} \sin^2 \mu_2 t\right) P(n_1, n_2) \tag{3.8}$$

Once the probability distribution function is known, the expectation value for the number of photons can be obtained through the relations

$$\langle \hat{n}_i(t) \rangle = \sum_{n_i} n_i P(n_1, n_2, t) \tag{3.9}$$

Thus we have:

$$\langle \hat{n}_1(t) \rangle^{gr} = \bar{n}_1 - k_1 \sum_{n_i} \nu_2 \frac{\sin^2 \mu_2 t}{\mu_2^2} P(n_1, n_2) \tag{3.10a}$$

$$\langle \hat{n}_2(t) \rangle^{gr} = \bar{n}_2 + k_2 \sum_{n_i} \nu_2 \frac{\sin^2 \mu_2 t}{\mu_2^2} P(n_1, n_2) \tag{3.10b}$$

By using the constant of motion operators (2.4) we have

$$\langle \hat{S}_{11}(t) \rangle^{gr} = \sum_{n_i} \nu_2 \frac{\sin^2 \mu_2 t}{\mu_2^2} P(n_1, n_2) \tag{3.11a}$$

$$\langle \hat{S}_{22}(t) \rangle^{gr} = \sum_{n_i} \left(\cos^2 \mu_2 t + \frac{E_2^2}{4\mu_2^2} \sin^2 \mu_2 t\right) P(n_1, n_2) \tag{3.11b}$$

It is easy to show that $\Sigma_1^2 \langle S_{jj}(t) \rangle = I$. We can also write the expectation value for the operator $\hat{a}_1^{S_1}(\hat{a}_2^+)^{S_2}$; for the ground state we find

$$\begin{aligned}
&\langle \hat{a}_1^{S_1}(\hat{a}_2^+)^{S_2} \rangle^{gr} \\
&= \exp\left\{-i\left[\left(w_1 + \frac{1}{2}\alpha_1\right)S_1 - \left(w_2 + \frac{1}{2}\alpha_2\right)S_2\right]t\right\} \\
&\times \sum \left\{\left[\cos t\mu_2(n_1 + S_1, n_2) - \frac{iE_2(n_1 + S_1, n_2)}{2\mu_2(n_1 + S_1, n_2)} \sin t\mu_2(n_1 + S_1, n_2)\right]\right. \\
&\times \left[\cos t\mu_2(n_1, n_2 + S_2) + \frac{iE_2(n_1, n_2 + S_2)}{2\mu_2(n_1, n_2 + S_2)} \sin t\mu_2(n_1, n_2 + S_2)\right] \\
&\left. + \nu_2(n_1, n_2 + S_2) \frac{\sin t\mu_2(n_1 + S_1, n_2)}{\mu_2(n_1 + S_1, n_2)} \frac{\sin t\mu_2(n_1, n_2 + S_2)}{\mu_2(n_1, n_2 + S_2)}\right\}
\end{aligned}$$

$$\times\ \rho_{n_1+S_1,n_2;n_1,n_2+S_2} \tag{3.12}$$

where $\rho_{n_1n_2m_1m_2} = \langle n_1, n_2 | \rho_F(0) | m_1, m_2\rangle$.

Finally, the expectation value for the operators $\hat{n}_1^{S_1}\hat{n}_2^{S_2}$ is of the form

$$\langle \hat{n}_1^{S_1}\hat{n}_2^{S_2}\rangle^{\text{gr}} = \overline{n_1^{S_1}n_2^{S_2}} + \sum_{n_i} [(n_1 - k_1)^{S_1}(n_2 + k_2)^{S_2} - n_1^{S_1}n_2^{S_2}]$$

$$\times\ \nu_2 \frac{\sin^2 \mu_2 t}{\mu_2^2} P(n_1 n_2) \tag{3.13}$$

where the bar denotes the initial value for the average.

3.2. The Atom in Its Excited State

By using the field density matrix (3.7) in the expression for the density matrix of the upper state (3.5b), we obtain the probability distribution function

$$P^{\text{ex}}(n_1, n_2, t) = \left(\cos^2 \mu_1 t + \frac{E_1^2}{4\mu_1^2} \sin^2 \mu_1 t\right) P(n_1, n_2)$$

$$+\ \nu_2 \frac{\sin^2 \mu_2 t}{\mu_2^2} P(n_1 - k_1, n_2 + k_2) \tag{3.14}$$

We can therefore obtain the expectation value $\langle \hat{n}_i(t)\rangle^{\text{ex}}$ through the relation (3.9),

$$\langle \hat{n}_1(t)\rangle^{\text{ex}} = n_1 + k_1 \sum_{n_i} \left[\cos^2\mu_1 t + \frac{E_1^2 \sin^2\mu_1 t}{4\mu_1^2}\right] P(n_1, n_2) \tag{3.15a}$$

$$\langle \hat{n}_2(t)\rangle^{\text{ex}} = n_2 - k_2 \sum_{n_i} \nu_1 \frac{\sin^2\mu_1 t}{\mu_1^2} P(n_1, n_2) \tag{3.15b}$$

Using (2.4), we obtain the formulas for the occupation numbers in the atomic levels as

$$\langle \hat{S}_{11}(t)\rangle^{\text{ex}} = \sum_{n_i} \left[\cos^2\mu_1 t + \frac{E_1^2}{4\mu_1^2} \sin^2\mu_1 t\right] P(n_1, n_2) \tag{3.16a}$$

$$\langle \hat{S}_{22}(t)\rangle^{\text{ex}} = \sum_{n_i} \nu_1 \frac{\sin^2\mu_1 t}{\mu_1^2} P(n_1, n_2) \tag{3.16b}$$

Also we can show that $\Sigma_1^2 \langle \hat{S}_{ii}(t)\rangle = I$.

However, the expectation value for $\langle \hat{a}_1^{S_1}(\hat{a}_2^+)^{S_2}\rangle$ is given by

$$\langle \hat{a}_1^{S_1}(\hat{a}_2^{+})^{S_2}\rangle^{ex}$$

$$= \exp\left\{i\left[\left(w_1 + \frac{1}{2}\alpha_1\right)S_1 - \left(w_2 + \frac{1}{2}\alpha_2\right)S_2\right]t\right\}$$

$$\times \left\{\left[\cos t\mu_1(n_1 + S_1, n_2) - \frac{iE_1(n_1 + S_1, n_2)}{2\mu_1(n_1 + S_1, n_2)} \sin t\mu_1(n_1 + S_1, n_2)\right]\right.$$

$$\times \left[\cos t\mu_1(n_1, n_2 + S_2) + \frac{iE_1(n_1, n_2 + S_2)}{2\mu_1(n_1, n_2 + S_2)} \sin t\mu_1(n_1, n_2 + S_2)\right]$$

$$\left. + \nu_1(n_1 + S_1, n_2) \frac{\sin t\mu_1(n_1 + S_1, n_2)}{\mu_1(n_1 + S_1, n_2)} \frac{\sin t\mu_1(n_1, n_2 + S_2)}{\mu_1(n_1, n_2 + S_2)}\right\}$$

$$\times \rho_{n_1+S_1,n_2,n_1,n_2+S_2} \tag{3.17}$$

Finally, we have

$$\langle \hat{n}_1^{S_1}\hat{n}_2^{S_2}\rangle^{ex} = \overline{n_1^{S_1}n_2^{S_2}} + \sum_{n_1} [(n_1 + k_1)^{S_1}(n_1 - k_2)^{S_2} - n_1^{S_1}n_2^{S_2}]$$

$$\times \nu_1 \frac{\sin^2\mu_1 t}{\mu_1^2} P(n_1 n_2) \tag{3.18}$$

We note that in equation (3.17), if we take the complex conjugate and interchange the subscripts 1 and 2, we can easily obtain (3.12). Also by interchanging the subscripts 1 and 2, equations (3.10) and (3.11) become (3.15) and (3.16), respectively.

The cross correlation between modes 1 and 2 can be written as

$$\Delta_{\text{cross}} = \langle n_1(t)n_2(t)\rangle - \langle n_1(t)\rangle\langle n_2(t)\rangle \tag{3.19}$$

The two modes are said to be correlated if Δ_{cross} is positive and anticorrelated if Δ_{cross} is negative.

When we use the appropriate quantities of (3.17) and (3.12) and the expression for the second-order correlation function

$$g_i^{(2)}(t) = \frac{\langle n_i^2(t)\rangle - \langle n_i(t)\rangle}{\langle n_i(t)\rangle^2} \tag{3.20}$$

we can discuss the bunching and antibunching. Also, we can obtain the dipole moment operators as follows:

$$\langle \hat{S}_{12}(t)\rangle^{\text{gr}} = i\lambda e^{i(Z_1-Z_2)t} \sum_{n_i} \left[\frac{(n_1+k_1)!(n_2+k_2)!}{n_1!\, n_2!}\right]^{1/2}$$
$$\times \left[\cos t\mu_2(n_1, n_2+k_2)\right.$$
$$\left. + \frac{iE_2(n_1, n_2+k_2)}{2\mu_2(n_1, n_2+k_2)} \sin t\mu_2(n_1, n_2+k_2)\right]$$
$$\times \frac{\sin t\mu_1(n_1, n_2+k_2)}{\mu_1(n_1, n_2+k_2)}\, \rho_{n_1,n_2+k_2,n_1+k_1,n_2} \tag{3.21}$$

and

$$\langle \hat{S}_{12}(t)\rangle^{\text{ex}} = -i\lambda e^{i(Z_1-Z_2)t} \sum_{n_i} \left[\frac{(n_1+k_1)!(n_2+k_2)!}{n_1!\, n_2!}\right]^{1/2}$$
$$\times \left[\cos t\mu_1(n_1+k_1, n_2)\right.$$
$$\left. + \frac{iE_1(n_1+k_1, n_2)}{2\mu_1(n_1+k_1, n_2)} \sin t\mu_1(n_1+k_1, n_2)\right]$$
$$\times \frac{\sin t\mu_2(n_1+k_1, n_2)}{\mu_2(n_1+k_1, n_2)}\, \rho_{n_1,n_2+k_2,n_1+k_1,n_2} \tag{3.22}$$

While the dipole–dipole correlation functions take the form

$$\langle \hat{S}_{12}(t)\hat{S}_{21}(t')\rangle^{\text{gr}}$$
$$= \exp\left\{i\left[\left(w_1 + \frac{1}{2}\alpha_1\right)k_1 - \left(w_2 + \frac{1}{2}\alpha_2\right)k_2\right](t-t')\right\}$$
$$\times \sum_{n_i} \nu_2 \frac{\sin \mu_2 t \sin \mu_2 t'}{\mu_2^2} \left[\nu_2' \frac{\sin \mu_2' t \sin \mu_2' t'}{\mu_2^2}\right.$$
$$\left. + \left(\cos \mu_2' t + \frac{iE_2'}{2\mu_2'} \sin \mu_2' t\right)\left(\cos \mu_2' t' - \frac{iE_2'}{2\mu_2'} \sin \mu_2' t'\right)\right]$$
$$\times P[n_1, n_2] \tag{3.23}$$

with

$$f'(n_1, n_2) = f(n_1 - k_1, n_2 + k_2)$$

and

$$\langle \hat{S}_{12}(t)\hat{S}_{21}(t')\rangle^{ex}$$
$$= \exp\left\{i\left[\left(w_1 + \frac{1}{2}\alpha_1\right)k_1 - \left(w_2 + \frac{1}{2}\alpha_2\right)k_2\right](t - t')\right\}$$
$$\times \sum_{n_i}\left[\left(\cos\mu_1 t + \frac{iE_1}{2\mu_1}\sin\mu_1 t\right)\left(\cos\mu_1 t' - \frac{iE_1}{2\mu_1}\sin\mu_1 t'\right)\right]$$
$$\times \left[\left(\cos\mu_2 t + \frac{iE_2}{2\mu_2}\sin\mu_2 t\right)\left(\cos\mu_2 t' - \frac{iE_2}{2\mu_2}\sin\mu_2 t'\right)\right.$$
$$\left. + \nu_2\frac{\sin\mu_2 t\sin\mu_2 t'}{\mu_2^2}\right]P[n_1, n_2] \tag{3.24}$$

we note that equations (3.8)–(3.24) are obtained above if we put $\hat{E}_j = \Delta$, i.e., $\alpha_j = 0$.

3.3. The Spectrum

The emission spectrum is given by the Fourier transformation of the dipole–dipole correlation function weighted by the detector response function

$$\langle\psi|S_{12}(t_1)S_{21}(t_2)|\psi\rangle \tag{3.25}$$

with

$$|\psi\rangle = \langle n_1, n_2|\alpha_1, \alpha_2\rangle\left(\cos\frac{\theta}{2}|n_1, n_2, 1\rangle\right.$$
$$\left. + e^{-i\phi}\sin\frac{e}{2}|n_1, n_2, 2\rangle\right) \tag{3.26}$$

The transient spectrum is given through the relation (Agarwal and Puri, 1986; Eberly and Wodkiewicz, 1977; Zaheer and Zubairy, 1989)

$$S(w) = 2\Gamma\int_0^T dt_1\int_0^T dt_2\, e^{-(\Gamma - iw)(T-t_1)-(\Gamma+iw)(T-t_2)}$$
$$\times \langle\psi|S_{12}(t_1)S_{21}(t_2)|\psi\rangle \tag{3.27}$$

where T is the interaction time and $1/\Gamma$ is the detector's response time.

After some manipulations, we obtain the following formula for the transient spectrum:

$$
\begin{aligned}
&\frac{8}{\Gamma} S(w) \\
&\quad = |\langle\alpha_1, \alpha_2|n_1, n_2\rangle|^2 \cos^2\frac{\theta}{2}\left[\frac{\nu_2}{\mu_n'^2} LL^* + MM^*\right] \\
&\qquad + \langle\alpha_1, \alpha_2|n_1 + k_1, n_2 + k_2\rangle\langle n_1, n_2|\alpha_1, \alpha_2\rangle \\
&\qquad \times e^{-i\phi} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\left[\frac{-\nu_2\sqrt{\nu_1}}{\mu_n'^2\mu_n} LN^* - \frac{\sqrt{\nu_1}}{\mu_n} MK^*\right] \\
&\qquad + \langle\alpha_1, \alpha_2|n_1, n_2\rangle\langle n_1 + k_1, n_2 + k_2|\alpha_1, \alpha_2\rangle e^{i\phi} \sin\frac{\theta}{2}\cos\frac{\theta}{2} \\
&\qquad \times \left[\frac{-\nu_2\sqrt{\nu_1}}{\mu_n'^2\mu_n} L^*N - \frac{\sqrt{\nu_1}}{\mu_n} M^*K\right] \\
&\qquad + |\langle\alpha_1, \alpha_2|n_1 + k_1, n_2 + k_2\rangle|^2 \\
&\qquad \times \sin^2\frac{\theta}{2}\left[\frac{\nu_1\nu_2}{\mu_n'^2\mu_n^2} NN^* + \frac{\nu_1}{\mu_n^2} kk^*\right]
\end{aligned} \tag{3.28}
$$

where

$$
\begin{aligned}
L &= -E_1^+F(\mu_n, \mu_n') - E_1^-F(-\mu_n, \mu_n') \\
&\quad + E_1^+F(\mu_n, -\mu_n') + E_1^-F(-\mu_n, -\mu_n') \\
M &= E_1^+E_2^+F(\mu_n, \mu_n') + E_1^-E_2^+F(-\mu_n, \mu_n') \\
&\quad + E_1^+E_2^-F(\mu_n, -\mu_n') + E_1^-E_2^-F(-\mu_n, -\mu_n') \\
N &= F(\mu_n, \mu_n') - F(-\mu_n, \mu_n') - F(\mu_n, -\mu_n') + F(-\mu_n, -\mu_n') \\
K &= -E_2^+F(\mu_n, \mu_n') + E_2^+F(-\mu_n, \mu_n') \\
&\quad - E_2^-F(\mu_n, -\mu_n') + E_2^-F(-\mu_n, -\mu_n')
\end{aligned}
$$

$$
F(\mu_n, \mu_n') = \frac{\exp\{i[\mu_n + \mu_n' - (w - \Sigma)]T\} - \exp\{-\Gamma T\}}{\Gamma + i[\mu_n + \mu_n' - (W - \Sigma)]} \tag{3.29a}
$$

$$
E_j^{\pm} = 1 \pm \frac{E_j}{2\mu_j}, \qquad \Sigma = Z_2 - Z_1,
$$

$$
\mu_n = \mu_1, \qquad \mu_n' = \mu_2 \tag{3.29b}
$$

Note that if we take $k_1 = 1$, $k_2 = 0$, $\alpha_j = 0$, and $\Delta = 0$ we obtain the spectrum obtained earlier (Zaheer and Zubairy, 1989).

4. CONCLUSIONS

In this paper, we considered a generalized multiphoton nonlinear Jaynes–Cummings model (JCM). We further assumed an intensity-dependent level shift. The detuning parameter in this case differed from the one obtained before since it depends on the photon number operator. We solved the model exactly and found the time evolution operator, and hence calculated the probability distribution functions and some statistical aspects for the initial photon states. The dipole moment as well as the dipole–dipole correlation functions were obtained. We obtained the general form of the transient spectrum established earlier as a special case. It is concluded that the former studies can be considered here; for example, by putting $k_1 = 1$, $k_2 = 0$, and $\alpha_j = 0$ we get the original JCM; by putting $\alpha_j = 0$ we get the model studied by Abdalla *et al.* (1990, 1991). The model of Sukumar and Buck (1984) is obtained by putting $k_2 = 0$. Therefore, this model represents a more generalized JCM than studied before.

ACKNOWLEDGMENTS

The author thanks Prof. A.-S. F. Obada for his helpful remarks and guidance throughout this work.

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